correlation is obtained between theory and experimental data More experimental studies are necessary to confirm the validity of the predictions in the short cylinder range, even though the present results have been shown in this range to reduce to the classical predictions when (b/L) approaches unity

Before concluding, it is worthwhile to reiterate that the present investigation was restricted to a circumferential band loading that was uniformly distributed and equally spaced $(L/2)$ along the generator of the cylinder The analysis presented in Ref 1 is, however, sufficiently general so as to permit an extension of the theoretical results to circumferential band loadings, which vary along the generator of a simply supported cone or cylinder with arbitrary spacings The extension is straightforward and only involves a simple integration procedure once the appropriate step function has been determined to describe the loading condition under consideration

References

[1] Serpico, J C , "Elastic stability of orthotropic conical and cylindrical shells subjected to axisymmetric loading conditions," AIAA J **1**, 128–137 (1963)

[2] Almroth, B O and Brush D O Buckling of a finite length cylindrical shell under a circumferential band pressure," J Aerospace Sci **28**, 573–592 (1961)

[3] Batdorf, S B , 'A simplified method of elastic stability analysis for thin cylindrical shells," NACA Rept 874 (1947)

Laminar Compressible Mixing behind Finite Bases

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In Ref 1 Chapman considered the problem of laminar mixing at constant pressure for a fluid with Prandtl number one, and a viscosity power law of the form $\mu \sim T^{\omega}$ Because of the recent interest in the fluid-mechanic description of hypersonic wakes, mixing problems are being again investigated intensely both experimentally and theoretically As an example, Ref 2 extends Ref 1 by assuming a Blasius starting velocity profile rather than a uniform one (A uniform profile was assumed by Chapman as a necessity imposed for the conservation of similarity)

It will be shown in this brief note that the velocity along the dividing streamline can be determined in a simple manner by approximating the integral solution of Ref 1 The effect of finite base radius (or Reynolds number) is also investigated using this approximate solution

In terms of the stream function ψ, the differential equation of motion is

$$\frac{\zeta}{2}\frac{du}{d\zeta} + \frac{d}{d\zeta}\left(g\frac{du}{d\zeta}\right) = 0 \qquad (1)$$

where

$$\zeta = \psi/(U\nu sC)^{1/2} \qquad g(\zeta) = uT^{\omega-1}$$
$$T = T_d - [(\gamma - 1)/2]\, M^2u^2 + (T_0 - T_d)u$$

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T and u are nondimensional with respect to the freestream, the subscripts d and 0 refer to the "dead water" region and stagnation point, and M is the freestream Mach number Chapman's boundary conditions are as follows:

$$\text{at } \zeta = \infty: \ u = 1 \qquad \zeta = -\infty: \ u = 0 \qquad (2)$$

The velocity at the dividing streamline u_D is given after a formal solution of Eq (1) as follows:

$$u_D = \int_{\infty}^{0} F d\zeta \Big/ \left(\int_{-\infty}^{0} F d\zeta + \int_{0}^{+\infty} F d\zeta\right) \qquad (3)$$

where

$$F(\zeta) = \exp\left\{-\int_0^{\zeta} \frac{\zeta}{2g}\, d\zeta\right\}\Big/ g \qquad (4)$$

We make the observation that, in the expression for the integrand F, the contribution of the exponential part in the numerator is stronger than the denominator for increasing g (or ζ) Hence, in the interval I, $-\infty < \zeta \leq 0$, and in the interval II, $0 \leq \zeta < +\infty$, most of the contribution comes about from the values of g corresponding to the highest ζ inside the interval For a first iteration let us, therefore, assume for u the following step function: inside I, $u = u_D$, inside II, $u = 1$ Simple integration of Eq (3) yields

$$u_D = \frac{1}{1 + (u_D T_D^{\omega-1})^{1/2}} \qquad (5)$$

Assuming that $T_d = T_0$ and $\omega = 0\ 75$ the calculations show that, for $M = 0, 1, 2, 3, 4, 5, 7, 10, 15, 20$, the corresponding values of u_D are 0 570, 0 573, 0 581, 0 590, 0 598, 0 605, 0 619, 0 634, 0 652, 0 664 For $M = 0$ and 5, Chapman[3] gives, through an exact numerical solution, $u_D = 0\ 587$ and 0 597 Comparison shows that our closed-form approximation is in error of less than -3% and $+1\ 5\%$, correspondingly

Experiments show[4] that u_D is a function of Reynolds number The analysis of Ref 2, in which the influence of an initial finite boundary-layer thickness was studied, through the parameter $s^* \sim s/s_b$ yields results that are independent of the base radius r_0 (This occurs because, as can be seen from Fig 1, $r_0 = s_b \sin\alpha = s_n \sin\beta$) One might conjecture that one way to introduce the Reynolds number would be to assume that $u = 0$ not at $\zeta = -\infty$ but at $\zeta = -\zeta_n$, where ζ_n is finite and positive This assumption implies a finite radius in the direction perpendicular to the main flow Following the same method used for the derivation of Eq (5), we find

$$u_D = \operatorname{erf}(\chi_n)/[\operatorname{erf}(\chi_n) + (g_D)^{1/2}] \qquad (6)$$

where $\chi_n = \zeta_n/2\,(g_D)^{1/2}$, and erf denotes the error function Figure 1 shows the function $u_D(M,\zeta_n)$ for three different Mach numbers

These results are best interpreted in the physical plane s,y For simplicity assume $M = 0$, so that[1]

$$y\left(\frac{u}{\nu s}\right)^{1/2} = \int_0^{\zeta} \frac{d\zeta}{u} \qquad (7)$$

Let ζ_n correspond to y_n and s_n, where the subscript n indicates the position of the "neck" of thickness h as shown† in Fig 2 To calculate the integral in Eq (7), we approximate $u(\zeta)$ in the interval $0 \leq \zeta \leq \zeta_n$ by dropping the first term in Eq (1) A simple integration yields

$$u = u_D\, [1 - (\zeta/\zeta_n)]^{1/2} \qquad (8)$$

Introducing the foregoing into Eq (7), we have

$$y_n(u/\nu s_n)^{1/2} = 2(\zeta_n/u_D) \qquad (9)$$

† A correction of this geometry to take into account the weaker growth of the boundary layer above the dividing streamline is very small

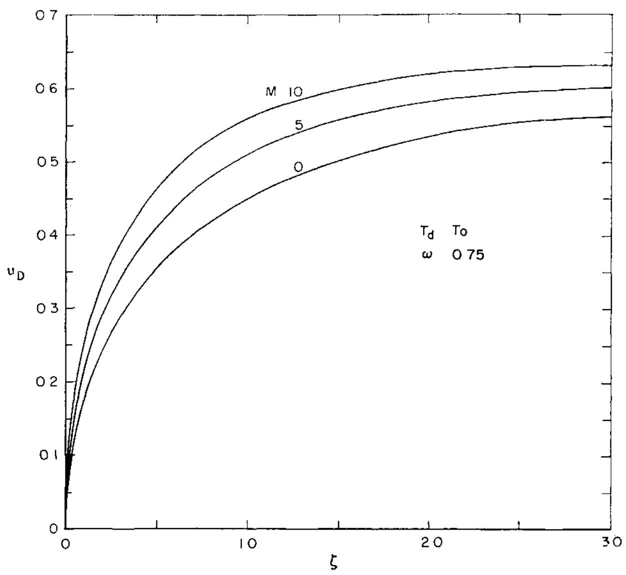

Fig 1 The velocity at the dividing streamline for different boundary conditions inside the base region

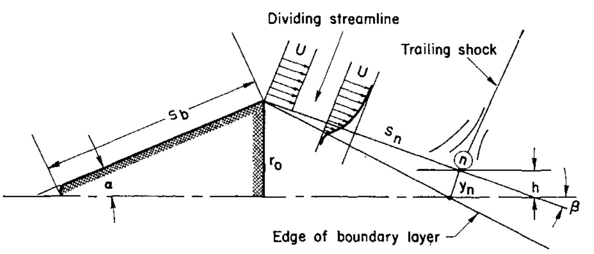

Fig 2 Schematic diagram of base flow

Assuming that all of the mass contained in the boundary layer over the body goes through the neck, we set

$$h/r_0 = 40(Ur_0/\nu)^{1/2} \tag{10}$$

The factor 40 is fixed by the experiments of Ref 3 Setting the position of the neck λ radii back of the base, Eq (9) yields, after some trigonometry,

$$\zeta_n = 20\, u_D/(\lambda \cos\beta)^{1/2} \tag{11}$$

From this equation, it appears that the value of u_D is again directly independent of the Reynolds number Since the angle β is of the order of 10°, so that $\cos\beta \approx 1$, and since from experiments λ is of order one, the assumption $\zeta_n \rightarrow \infty$ is still reasonable The proof of this last statement lies in the fact that Eq (11) yields a value of $\zeta_n = 12$ when $u_D \approx 0\,6$, $\lambda \approx 1$, and $\cos\beta \sim 1$; then, Fig 1 indicates that our choice for u_D is consistent with the fact that at $\zeta_n = 12$ the asymptotic value for u_D has been reached In fact, it is reached roughly when $\zeta_n > 3\,0$ One needs a neck length of the order of 10 radii in order to make a correction in u_D for finite base radius

References

[1] Chapman, D R , 'Laminar mixing of a compressible fluid," NACA Rept 958 (1950)

[2] Denison, M R and Baum, E , Compressible free shear layer with finite initial thickness," AIAA J 1, 342–349 (1963)

[3] Chapman, D R , Kuehn, D M , and Larson, H K , "Investigation of separated flows in supersonic and subsonic streams with emphasis on the effect of transistion," NACA Rept 1356 (1958)

[4] Dewey, C F , Jr , "Measurements in highly dissipative regions of hypersonic flows, Part II: The near wake of a blunt body at hypersonic speeds," Ph D Thesis, Calif Inst Tech (1963)

Acoustic Absorption Coefficients of Combustion Gases

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Nomenclature

c = speed of sound, cm sec^{-1}
C_p = specific heat at constant pressure, cal g^{-1} °K^{-1}
C_v = specific heat at constant volume, cal g^{-1} °K^{-1}
f = frequency, sec^{-1}
f_n = frequency of the nth harmonic, sec^{-1}
l = length of resonance tube, cm
n = harmonic index, integer
P = pressure, dyne cm^{-2}
P_r = Prandtl number, dimensionless
R = tube radius, cm
γ = ratio of specific heats
δ = frequency width of a resonance peak, sec^{-1}
λ = thermal conductivity, cal cm^{-1} sec^{-1} °K^{-1}
μ = dynamic viscosity, poise
σ = wall acoustic damping coefficient, cm^{-1}

WORK on the problem of combustion instability in solid propellant rocket motors has focused interest upon the various acoustic losses and gains that are present in these motors

Acoustic damping in the combustion gas phase is one of the acoustic energy sinks that is present in any rocket motor, and it is the purpose of this note to present some initial results from an experimental investigation of the acoustic damping constant of solid propellant combustion gases The data of this note apply to combustion gases that have been cooled to room temperature

Experimental Apparatus

The experiments were carried out in an acoustic resonance tube patterned after the one used by Parker [1] A Goodman's VR-II electromagnetic shaker driver was used to drive a rigid flat-topped piston that closed one end of a 50 mm diam steel tube The other end of the tube was closed by a rigid brass plate which carried a flush-mounted condenser microphone at its center The tube length between the closures was 683 mm

During an experiment, the tube was filled with the test gas at 1 atm pressure and room temperature, and the apparatus was set to drive the piston at a constant velocity amplitude The driving frequency was varied from the fundamental of the tube up through the 12th harmonic, which represented the high-frequency limit of the driving apparatus All frequencies were determined to ±0 1 cps by an electronic counter

The resonant frequencies of the tube were given by

$$f_n = n(c/2l) \tag{1}$$

and the acoustic damping coefficient (wall damping was the only significant gas phase damping mechanism involved) was obtained from the relation[1 2]

$$\sigma = \pi\delta/c \tag{2}$$

The frequency width of a resonant peak, δ, was determined as the difference in frequencies on a given peak at the points

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